Ch. 6 to 7.3 Review 2

1. Determine the number of terms in the geometric sequence: $\frac{1}{128}, \frac{1}{32}, \frac{1}{8}, \ldots, 2048$

$r = \frac{\frac{1}{32}}{\frac{1}{128}} = 4$

$t_n = t_1(r)^{n-1}$

$2048 = \frac{1}{128}(4)^{n-1}$

$262144 = 4^{n-1}$

$4^9 = 4^{n-1}$

$9 = n-1$

$n = 10$

2. A new well produces 48000 L of water in the first month. If the volume of water pumped decreases by 6% each month, determine the total volume of water, in litres, that will be pumped from the well before it runs dry.

$r = 0.94$

$S_\infty = \frac{t_1}{1-r} = \frac{48000}{1-0.94} = 800{,}000\text{L}$

3. In a geometric series, if the sum of the first 12 terms is 20475 and the common ratio is 2, determine the value of the first term.

$S_n = \frac{t_1(1-r^n)}{1-r}$

$20475 = \frac{t_1(1-2^{12})}{1-2}$

$-20475 = t_1(1-4096)$

$-20475 = -4095t_1$

$t_1 = 5$

4. Evaluate: a) $\sum_{k=5}^{13} 3(2)^{k-2}$

$t_1 = 3(2)^{5-2} = 24$

$t_2 = 3(2)^{6-2} = 48$

$r = \frac{48}{24} = 2$, $n = 13-5+1 = 9$

$S_9 = \frac{24(1-2^9)}{1-2} = 12264$

b) $\sum_{k=1}^{\infty} \frac{1}{3^k}$

$t_1 = \frac{1}{3^1} = \frac{1}{3}$

$t_2 = \frac{1}{3^2} = \frac{1}{9}$

$r = \frac{1}{9} \div \frac{1}{3} = \frac{1}{3}$

$S_\infty = \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{1}{2}$

5. Laura invests in a bond which pays interest at the rate of 2.5% per year compounded weekly. After 10 years the value of the bond has increased to $1267.28. What was the original value of the bond?

$A = A_0\left(1+\frac{i}{n}\right)^{nt}$

$1267.28 = A_0\left(1+\frac{0.025}{52}\right)^{52(10)}$

$1267.28 = A_0(1.284)$

$A_0 = \$987.02$

6. Mr. H worked for SD41 for 5 years. His annual salary was $38000 during his first year. Each year his salary increased by 2% over the previous year's salary. Suppose he was able to keep half of his salary for a house he would like to purchase that requires a down payment of $100,000. Can he afford it?

$S_5 = \frac{38000(1-1.02^5)}{1-1.02}$

$= \$197{,}753.53$

$\$197{,}753.53 \div 2$

$= \$98{,}876.76.$

His dream is shattered :(

7. Simplify: $216^x \div (1296^{5x-4} \times 36^{x+5})$

$$\frac{(6^3)^x}{(6^4)^{5x-4} \cdot (6^2)^{x+5}} = \frac{6^{3x}}{6^{20x-16} \cdot 6^{2x+10}} = \frac{6^{3x}}{6^{22x-6}} = \boxed{6^{-19x+6}}$$

8. Solve: a) $\left(\frac{1}{4}\right)^{x-12} = 32^{2x+5}$

$(2^{-2})^{x-12} = (2^5)^{2x+5}$

$2^{-2x+24} = 2^{10x+25}$

$-2x+24 = 10x+25$

$-12x = 1$

$\boxed{x = \frac{-1}{12}}$

b) $\sqrt[4]{\frac{25^{x-1}}{125^{3x+2}}} = 5^x(625^{x-2})$

$\left(\frac{(5^2)^{x-1}}{(5^3)^{3x+2}}\right)^{\frac{1}{4}} = (5^x)(5^4)^{x-2}$

$\left(\frac{5^{2x-2}}{5^{9x+6}}\right)^{\frac{1}{4}} = (5^x)(5^{4x-8})$

$(5^{-7x-8})^{\frac{1}{4}} = 5^{5x-8}$

$5^{\frac{-7}{4}x-2} = 5^{5x-8}$

$\frac{-7}{4}x - 2 = 5x - 8$

$-\frac{27}{4}x = -6$

$-27x = -24$

$x = \frac{24}{27} = \boxed{\frac{8}{9}}$

9. Graph $y = -2(4^{-2x+6}) - 1$

$y = 4^x$

x	y
-1	1/4
0	1
1	4

$y = -2(4^{-2(x-3)}) - 1$ (a = -2, b = -2, c = 3, d = -1)

$\frac{x}{-2} + 3, \; -2y - 1$

x	y
3.5	-1.5
3	-3
2.5	-9

H.A: $y = -1$

D: $x \in \mathbb{R}$

R: $y < -1$

y-int:

$y = -2(4^{-2(0)+6}) - 1$

$= -8193$