Fun problems

1. The digit product of 234 is $2 \times 3 \times 4 = 24$. Other numbers also have a digit product of 24. For example, 2223, 113181 and 38 each have a digit product of 24. The number 38 is the smallest positive integer with a digit product of 24. Determine the smallest positive integer who digit product is 2000.

$2000 = 2^4 \times 5^3$

$2^4 \downarrow 16$, $5^3 \downarrow 125$

Smallest # to have a DP 16 is 2x8.

" 125 5x5x5

∴ The smallest combination is

$\boxed{25558}$

2. Some tokens are placed in a bag. Each of the tokens has a positive integer stamped on one of its sides. It is possible that more than one token in the bag has the same number stamped on it. The average of all the numbers stamped on the tokens in the bag is 56. If a token with the number 68 on it is removed from the bag, the average of the numbers stamped on the remaining tokens is 55. Determine the largest possible integer that could appear on one of the tokens in the bag.

Let x be the # of tokens in the bag.

$56x - 68 = 55(x-1)$

$56x - 68 = 55x - 55$

$x = 13$

Total: $56 \times 13 = 728$

1 token = 68, we want the biggest possible

∴ 11 of them will be 1

∴ $728 - 68 - 11(1)$

$\boxed{= 649}$

3. The captain of a cruise was asked by Mr. H about how many guests were on the cruise. The captain replied: $\frac{1}{6}$ of our guests are seniors, $\frac{1}{4}$ of our guests are children or teenagers, there are three times as many adults as teenagers, and there are 138 children on board. Mr. H is clueless again, can you help him by finding out how many passengers are on board?

Senior: $\frac{1}{6}$

$C + T = \frac{1}{4}$

∴ Adult $= 1 - \frac{1}{6} - \frac{1}{4}$

$= \frac{7}{12}$

Adult $= 3 \times T$

$\frac{7}{12} = 3T$

$T = \frac{7}{36}$

∴ $C + \frac{7}{36} = \frac{1}{4}$

$C = \frac{1}{4} - \frac{7}{36}$

$= \frac{2}{36} = \frac{1}{18}$

$\frac{1}{18}$ Total $= 138$

∴ Total

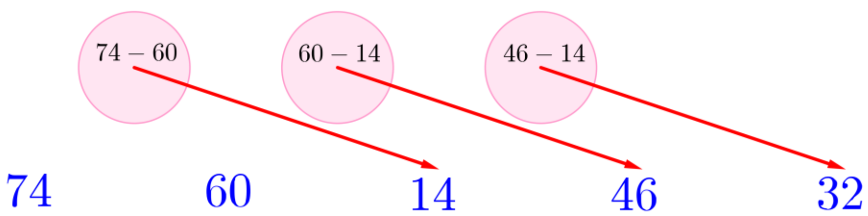
$= 138 \times 18$

$\boxed{= 2484}$

4.

In the sequence 74, 60, 14, 46, 32, $\cdots$, each number after the second number is obtained by finding the non-negative difference between the previous 2 numbers. The diagram below illustrates how each term after the second term is derived.

Determine the sum of the first 1300 numbers in the sequence.



$1300 - 14$

$= 1286$

$1286 \div 3$

$= 428$ R2

$+6$

$74\times 2 \quad + \quad 46\times 2 \quad + \quad 18\times 2 \quad + \quad 10\times 2$

$\underline{74+60+14}+\underline{46+32+14}+\underline{18+4+14}+\underline{10+4+6}+\underline{2+4}+2+\underline{2+0}$

$\underline{+2+2+0}+2\ \underline{+2+0+}\ldots$

$(74+46+18+10)\times 2 + 6 + 429\times 4$

$= \boxed{2018}$

5. The product of the positive integers 1 to 6 is $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ or we can write in an abbreviated form as "6!" or 6 factorial. For a positive integer n, the product of the positive integers from 1 to n is n!. Find the smallest possible value of n so that n! ends in exactly six zeros.

N = 25

$1! = 1$

$2! = 2$

$3! = 6$

$4! = 24$

$5! = 120$

$\vdots$

$10! = 3628800$

$\vdots$

$15! = \quad 000$

$20! = \quad 0000$

$25! = \ldots 000000$

$\ldots\times 5$

$\ldots\times(2\times 5)$

$\ldots\times(3\times 5)$

$\ldots\times(4\times 5)$

$\ldots\times(5\times 5)$

because $25 = 5\times 5$

So we multiply 5 twice

$\therefore$ increase the # of zeros twice!

$\boxed{n=25}$