

Foundations and Pre-Calculus Math 10	Unit
<b>Extension</b>	
By the end of this lesson I will be able to: <ul style="list-style-type: none"> <li>Solve systems of linear equations by using row reduction</li> </ul>	<b>7.F</b>

Is there a more efficient way of solving system of equations?

Ex.  $x + y + z = 3$   
 $x + 2y + 3z = 0$   
 $x + 3y + 4z = -2$

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 1 & 2 & 3 & 0 \\ 1 & 3 & 4 & -2 \end{bmatrix}$$

Step 1: Write this in an augmented matrix form.

Step 2: Row reduction

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & -3 \\ 0 & 2 & 3 & -5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -1 & 6 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & -1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$(5, -1, -1)$

This is what we called a reduced row echelon form.

$2x + y - 2z = 2$   
1.  $-x - 2y + 3z = 4$   
 $3x - y + 2z = 13$

$$\begin{bmatrix} 2 & 1 & -2 & 2 \\ -1 & -2 & 3 & 4 \\ 3 & -1 & 2 & 13 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 1 & -2 & 2 \\ 0 & -3 & 4 & 10 \\ 0 & 5 & -10 & -20 \end{bmatrix} \rightarrow \begin{bmatrix} 10 & 0 & 0 & 30 \\ 0 & -1 & 0 & 2 \\ 0 & 1 & -2 & -4 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & -2 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$(3, -2, 1)$

$$2x + 3y - 2z = -15$$

2.  $-2x + y + 3z = 8$   
 $x - 2y + z = 13$

$$\begin{bmatrix} 2 & 3 & -2 & -15 \\ -2 & 1 & 3 & 8 \\ 1 & -2 & 1 & 13 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 3 & -2 & -15 \\ 0 & 4 & 1 & -7 \\ 0 & 7 & 4 & -41 \end{bmatrix} \rightarrow \begin{bmatrix} 8 & 0 & -11 & -39 \\ 0 & 4 & 1 & -7 \\ 0 & 0 & 23 & 115 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 8 & 0 & -11 & -39 \\ 0 & 4 & 1 & -7 \\ 0 & 0 & 1 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 8 & 0 & 0 & 16 \\ 0 & 4 & 0 & -12 \\ 0 & 0 & 1 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$$(2, -3, 5)$$

3.  $a + 2b + 3c - 2d = 6$   
 $3a + 2b - c + 2d = 4$   
 $2a - b - 2c - 3d = 2$   
 $2a - 3b + 2c + d = 8$

$$\begin{bmatrix} 1 & 2 & 3 & -2 & 6 \\ 3 & 2 & -1 & 2 & 4 \\ 2 & -1 & -2 & -3 & 2 \\ 2 & -3 & 2 & 1 & 8 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 & -2 & 6 \\ 0 & 4 & 10 & -8 & 14 \\ 0 & 5 & 8 & -1 & 10 \\ 0 & 2 & -4 & -4 & -6 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & -2 & 2 & -1 \\ 0 & 2 & 5 & -4 & 7 \\ 0 & 0 & 18 & -36 & 30 \\ 0 & 0 & 9 & 0 & 13 \end{bmatrix} \rightarrow \begin{bmatrix} 9 & 0 & 0 & 18 & 17 \\ 0 & 18 & 0 & -36 & -2 \\ 0 & 0 & 9 & 0 & 13 \\ 0 & 0 & 0 & -36 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 9 & 0 & 0 & 0 & 19 \\ 0 & 18 & 0 & 0 & -6 \\ 0 & 0 & 9 & 0 & 13 \\ 0 & 0 & 0 & -9 & 1 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 19/9 \\ 0 & 1 & 0 & 0 & -1/3 \\ 0 & 0 & 1 & 0 & 13/9 \\ 0 & 0 & 0 & 1 & -1/9 \end{bmatrix}$$

$$\left( \frac{19}{9}, -\frac{1}{3}, \frac{13}{9}, -\frac{1}{9} \right)$$