7.4 – 7.9 Review 2

1. Solve for x:

a) $\log_5 x + \log_8 x = 4$

$$\frac{\log x}{\log 5} + \frac{\log x}{\log 8} = 4$$

$$\frac{\log 8 \log x + \log 5 \log x}{\log 5 \log 8} = 4$$

$$\log x(\log 5 + \log 8) = 4\log 5 \log 8$$

$$\log x = \frac{4\log 5 \log 8}{\log 40}$$

$$x = 10^{\frac{4\log 5\log 8}{\log 40}} = \boxed{37.67}$$

b) $\frac{1}{2}\log_4(x+4) + \frac{1}{2}\log_4(x-4) = \log_4 3$

$$\frac{1}{2}\left(\log_4(x+4) + \log_4(x-4)\right) = \log_4 3$$

$$\log_4[(x+4)(x-4)] = 2\log_4 3$$

$$\log_4[x^2-16] = \log_4 9$$

$$x^2 - 16 = 9$$

$$x^2 = 25$$

$$x = \overset{+}{\cancel{-}}5$$

c) $2^{3x} = 5^{3x+1}$

$$3x\log 2 = (3x+1)\log 5$$

$$3x\log 2 = 3x\log 5 + \log 5$$

$$3x\log 2 - 3x\log 5 = \log 5$$

$$x(3\log 2 - 3\log 5) = \log 5$$

$$\boxed{x = \frac{\log 5}{3\log 2 - 3\log 5}}$$

d) $3(2^{x+1}) = 6^x$

$$\log[3(2^{x+1})] = x\log 6$$

$$\log 3 + \log(2^{x+1}) = x\log 6$$

$$\log 3 + (x+1)\log 2 = x\log 6$$

$$\log 3 + x\log 2 + \log 2 = x\log 6$$

$$\log 3 - \log 2 = x\log 6 - x\log 2$$

$$\log\left(\frac{3}{2}\right) = x(\log 6 - \log 2)$$

$$x = \frac{\log\left(\frac{3}{2}\right)}{\log 6 - \log 2} = \boxed{\frac{\log\frac{3}{2}}{\log 3}}$$

2. Express $\log\frac{x^2}{10y^3}$ in terms of $\log x$ and $\log y$.

$$= \log x^2 - \log 10 - \log y^3$$

$$\boxed{= 2\log x - 1 - 3\log y}$$

3. Determine the Richter scale reading for an earthquake that is 5 times more intense than another earthquake that measures 4.0 on the Richter scale.

$$\frac{10^x}{10^4} = 5$$

$$10^x = 5 \cdot 10^4$$

$$x\log 10 = \log(50000)$$

$$\boxed{x = 4.7}$$

4. If $\log 5 = m \; and \; \log 3 = n$, then what is $\log 135$ in terms of $m \; and \; n$?

$\log 135$

$= \log(5 \times 3^3)$

$= \log 5 + \log 3^3$

$= \log 5 + 3\log 3$

$= \boxed{m + 3n}$

5. Determine the value of $\log_n(a^3 b)$ if $\log_n a = 3$ and $\log_n b = 4$

$\log_n(a^3 b)$

$= \log_n a^3 + \log_n b$

$= 3\log_n a + \log_n b$

$= 3(3) + 4$

$= 9 + 4$

$\boxed{= 13}$

6. Write $\log_2 x + \log_4 y$ as a single log.

$= \dfrac{\log x}{\log 2} + \dfrac{\log y}{\log 4}$

$= \dfrac{2\log x}{2\log 2} + \dfrac{\log y}{\log 4}$

$= \dfrac{2\log x}{\log 4} + \dfrac{\log y}{\log 4}$

$= \dfrac{2\log x + \log y}{\log 4}$

$= \dfrac{\log x^2 + \log y}{\log 4}$

$= \dfrac{\log x^2 y}{\log 4}$

$= \boxed{\log_4(x^2 y)}$

7. Simplify $\dfrac{\log_a x}{\log_{ab} x} - \dfrac{\log_a x}{\log_b x}$

$= \dfrac{\frac{\log x}{\log a}}{\frac{\log x}{\log ab}} - \dfrac{\frac{\log x}{\log a}}{\frac{\log x}{\log b}}$

$= \dfrac{\cancel{\log x}}{\log a} \times \dfrac{\log ab}{\cancel{\log x}} - \dfrac{\cancel{\log x}}{\log a} \times \dfrac{\log b}{\cancel{\log x}}$

$= \dfrac{\log ab}{\log a} - \dfrac{\log b}{\log a}$

$= \dfrac{\log ab - \log b}{\log a}$

$= \dfrac{\log\left(\frac{ab}{b}\right)}{\log a}$

$= \dfrac{\log a}{\log a}$

$= 1$